**Supplementary file**

# A. Supplementary formula

The line capacity constraints between flexible loads and distribution networks can be written as follows:

 (S1)

 (S2)

where *P*d2fe,cap represents the line capacity between the ferroalloy load and DN, *P*d2tcl,cap represents the line capacity between the TCLs and DN.

The line capacity constraints between flexible loads and wind farm can be expressed as:

 (S3)

 (S4)

 (S5)

where *P*i2fe,cap, *P*i2tcl,cap *P*i2mg,cap and represent the line capacity between the ferroalloy load, TCLs, MG and wind farm, respectively.

# B. Supplementary figures and parameters

Wind power output scenarios are shown in Fig. S1. The initial power and the adjustable range of TCLs are shown in Fig. S2. Table S1 shows the values of the other parameters in the case study.



**Fig. S1.** Wind power output scenarios.



**Fig. S2.** Initial power and the adjustable range of the TCLs.

## **Table S1** Values of the other parameters in case study

|  |  |
| --- | --- |
| Parameter | Value |
| , | 280 $/MWh |
|  | 28 $/MWh |
| *a*fe | 0.95 |
| *α*su, *α*in | 1% (wind restricted periods)  10% (wind-unrestricted periods) |
| *θ* | 0.9915 |
| *μ* | 0.001 |
|  | 56 $/MWh |
|  | 28.7 $/MWh |
|  | 0.7 $/MWh |
| , | 14 $/MWh, 175 $/MWh |
| *c*trfe | 130.67 $/(MW·day) |
| *k*fe | 0.5 |
| , | 46MW, 55MW |
|  | 50MW |
| , | 5MW, 7MW |
| *P*Tfe | 50MVA |
| *P*d2fe,cap,*P*d2tcl,cap | 46MW, 10MW |
| *P*i2fe,cap, *P*i2tcl,cap *P*i,mg,cap | 60MW, 10MW, 10MW |

# C. Steps for generating wind power output scenarios

This paper adopts the wind power output scenario generation method proposed in [R1]. Using this method, scenarios that incorporate the information of forecast error distribution and fluctuation distribution for short-term wind power can be generated. The specific steps are detailed below:

S1) Fit the forecast error distribution function

First, we statistically analyzed the day-ahead forecast and actual values of wind power within Ireland's power system in 2020 [R2], and converted the relevant data to the required scale (100 MW). Next, the wind power output forecast data were sorted. The range of 0-100 MW was divided into 20 power intervals (corresponding to 20 forecast bins, each with a width of 5 MW). The empirical cumulative distribution function was used to describe the forecast error distribution for wind power in each bin. For a given value *x*, the empirical cumulative distribution function is derived by summing the probabilities of all observations in the dataset that are less than *x*.

S2) Calculate the covariance matrix

This paper utilizes the covariance matrix ***G*** to reflect the correlation of wind power forecast errors across different time intervals, ensuring that randomly generated wind power output scenarios align with the statistical patterns of wind power fluctuations. The exponential covariance function [R3] is employed to calculate the covariance.

S3) Generate wind power output scenarios

After obtaining the covariance matrix ***G***, *S* random vectors ***Z*** are generated that follow a multivariate normal distribution *N*(***u***0, ***G***), where ***u***0, is a T-dimensional zero vector. By applying the inverse transformation to the random vectors ***Z***, wind power forecast error scenarios can be generated. These forecast error scenarios are then added to the predicted values to produce *S* wind power output scenarios.

# References

[R1]X. Ma, Y. Sun, H. Fang, Scenario generation of wind power based on statistical uncertainty and variability, IEEE T. Sustain. Energ. 4 (4) (2013) 894-904.

[R2]Annual renewable report. https://www.eirgrid.ie/grid/system-and-renewable-data-reports.

[R3]P. Pinson, R. Girard, Evaluating the quality of scenarios of short-term wind power generation, Appl. Energ. 96 (2012) 12-20.